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BASICS OF FINANCIAL ECONOMETRICS

EXAM REVISION NOTES

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MODULE I

INTRODUCTION TO FINANCIAL ECONOMETRICS

Financial Econometrics vs. Traditional Macro/Micro Econometrics: Although financial econometrics shares foundational estimation principles with traditional econometrics, the nature of financial market data creates profound structural divergences:

- **Observation Frequency:** ; Extremely High: Daily, Hourly, Minute-by-minute, and tick-by-tick real-time order book transactions.
- **Sample Sizes:** ; Massive (tens of thousands to millions of observations), providing immense statistical power.
- **Signal-to-Noise Ratio:** ; Extremely Low: Asset prices are dominated by stochastic noise, market sentiment, and arrival of unexpected news.
- **Distributional Properties:** ; Severely Non-Normal: Excess kurtosis (leptokurtosis), fat tails, skewness, and extreme jump risk.

Practical Applications in Modern Capital Markets: Financial econometrics serves as the quantitative backbone across four core domains of corporate finance and investment management:

- **2. Market Risk Management:** Quantitative measurement of downside portfolio exposure for regulatory capital compliance (Basel III/IV frameworks).
- **3. Derivative Valuation and Hedging:** Econometric calibration of historical and implied volatility surfaces used to price options, structured financial products, and credit default swaps, alongside calculating dynamic delta, gamma, and vega hedge ratios.
- 4.

Deterministic vs. Stochastic Relationships: , Total Revenue = Price * Quantity sold). In contrast, economic and financial behavior is inherently stochastic (probabilistic):

The Nature and Conceptual Origin of the Disturbance Term (u(i)): The error term is not merely a computational leftover; it is a fundamental theoretical necessity arising from four unavoidable realities of empirical observation:

- **Omission of Minor Explanatory Variables:** Economic theory emphasizes parsimony, focusing on primary causal drivers.
- Intrinsic Randomness in Human Market Behavior: Market participants do not behave like deterministic clockwork; emotional sentiment, panic, liquidity shocks, and cognitive biases inject genuine randomness into market pricing.
- **Measurement Errors in Empirical Data:** Economic data is subject to rounding, accounting classification differences, timing discrepancies, and survey response noise.
- **Mathematical Form Approximation:** Linear specifications approximate complex, potentially non-linear real-world relationships.

Typology of Financial Data Structures: The choice of econometric model and estimation technique is fundamentally governed by the structural dimension of the empirical data under investigation.

Time Series Data: Time series data consists of observations on a single economic unit (such as an individual stock, an index, a currency pair, or a sovereign interest rate) collected repeatedly across regular chronological intervals (daily, monthly, quarterly, or annually):

- **Mathematical Notation:** , T).
- **Core Characteristics:** The chronological sequence of observations is critical and cannot be arbitrarily shuffled.
- **Methodological Challenges:** Most financial time series in price levels exhibit non-stationarity (stochastic trends or random walks).

Pooled Cross-Sections and Panel / Longitudinal Data: When cross-sectional and time series dimensions are combined, researchers work with either pooled cross-sections or structured panel datasets:

Pooled Cross-Sections

Econometric Modeling of Panel Data: Fixed vs. Random Effects: , corporate culture, managerial capability, geographic location):

Temporal Frequency Dimensions and Market Microstructure: Financial econometricians analyze data across diverse temporal horizons:

- **Low Frequency (Monthly / Quarterly / Annual):** Ideal for long-term valuation metrics, macroeconomic factor pricing, corporate dividend policies, and structural accounting ratio analyses.
- **Daily Frequency:** The standard benchmark for modern asset pricing, mutual fund performance evaluation, beta estimation, and baseline market risk management.
- Intraday Ultra-High-Frequency (Tick-by-Tick) Data: High-frequency data captures every individual quote revision and trade execution timestamped to milliseconds or microseconds.

Simple Arithmetic Returns vs. Continuously Compounded Log Returns: Researchers calculate two primary forms of financial returns:

- **Cross-Sectional Aggregation:** ; Non-Linear: The log return of a portfolio is not the weighted Sum of of individual log returns: r(p) ≠ Sum(w(i) * r(i)).
- **Intertemporal Aggregation:** Multiplicative: Multi-period compounding requires multiplying gross returns: $(1 + R(0,T)) = (1 + R_1) * (1 + R_2) * ...$
- **Theoretical Range Bounds:** ; Unbounded in both directions: (-Infinity, +Infinity), matching the theoretical support of the Gaussian distribution.
- **Directional Symmetry:** 693.

Stylized Facts of Financial Time Series: Decades of empirical examination across global equity, bond, foreign exchange, and commodity markets have established a core set of statistical regularities known as the Stylized Facts of Asset Returns (Cont, 2001; Tsay, 2010; Brooks, 2019).

The Seven Universal Stylized Facts

- 1.
- **3. Gain / Loss Asymmetry (Skewness):** Financial asset returns typically display negative skewness (left-skewed distributions).
- **7. Volume-Volatility Correlation:** Daily trading volume is positively correlated with financial market volatility.

Computational Frameworks and Software Environments: Modern empirical financial econometrics relies on powerful statistical computing software to process massive financial datasets, execute econometric filters, and estimate non-linear parameters:

R Statistical Computing

EViews (Econometric Views)

Stata

Summary Synthesis

- **Financial Econometrics:** ; Essential for asset pricing, risk management (VaR), option pricing, and algorithmic trading.
- **Disturbance Term (u(t)):** ; Transforms deterministic economic theory into testable probabilistic econometric models.
- **Time Series Data:** ; Subject to autocorrelation and non-stationarity; requires unit root screening before modeling.
- **Panel Data:** ; Controls for unobserved individual heterogeneity using fixed and random effects models.

Advanced Analytical Dimensions and Applied Commerce Casework (Analytical Division 1): The concepts, statutory provisions, operational accounting rules, and quantitative models detailed in this module carry critical implications for enterprise governance, audited disclosure standards, and executive fiscal decision-making.

Comparative Regulatory and Practical Implementation Standards: A rigorous examination of corporate commercial practice highlights the indispensable role of standardized accounting principles.

MODULE II

STATISTICAL FOUNDATION OF FINANCIAL ECONOMETRICS

Probability Distributions and Statistical Moments in Finance: Financial asset returns are continuous random variables governed by probability density functions (PDFs). The probabilistic behavior of returns is characterized through four statistical moments.

Major Probability Distributions in Financial Econometrics: Four primary continuous probability distributions form the operational basis of econometric estimation and hypothesis testing:

- **Normal (Gaussian) Distribution:** Symmetric bell curve defined completely by Mean (mu) and Variance (sigma²).
- **Student's t-Distribution:** Symmetric bell shape governed by Degrees of Freedom (df = nu).
- **Chi-Square Distribution:** Sum of nu squared independent standard normal variables.
- **F-Distribution:** Ratio of two independent Chi-Square variables divided by their respective degrees of freedom: F(nu1, nu2).

Formal Test of Normality: The Jarque-Bera (JB) Statistic: Because financial returns deviate from normality, econometricians formally test whether empirical residuals or returns match a Gaussian distribution using the Jarque-Bera test:

Statistical Inference and Classical Hypothesis Testing: Statistical inference encompasses point estimation, confidence interval construction, and hypothesis testing.

Confidence Intervals and Standard Errors: A point estimate provides a single numerical value, but conveys no information regarding sampling precision. Econometricians construct Confidence Intervals (CI):

Type I Error (alpha)

Type II Error (beta) Statistical Power

The Classical Hypothesis Testing Triad: LR, Wald, and LM Tests: In advanced financial econometrics, testing non-linear parameter restrictions relies on three asymptotically equivalent estimation principles:

- **1. Likelihood Ratio (LR) Test:** Requires estimating both the unconstrained model and the constrained (restricted) model using Maximum Likelihood.
- **2. Wald Test:** Requires estimating only the unconstrained model.
- **3. Lagrange Multiplier (LM / Score) Test:** Requires estimating only the restricted model.
- **Finite-Sample Relationship:** In linear regression models with normal disturbances, a strict numerical inequality holds: Wald ≥ LR ≥ LM.

The Classical Linear Regression Model (CLRM) and OLS: The Classical Linear Regression Model estimated via Ordinary Least Squares (OLS) represents the foundational workhorse of empirical finance.

Precision and Sampling Variance of OLS Estimators: The precision of the OLS slope estimator depends on three fundamental factors:

- **Disturbance Variance (sigma²):** Var(b1) is directly proportional to error variance sigma² = RSS / (N - 2).
- **Variation in Independent Variable X:** Var(b1) is inversely proportional to Sum[(X(i) - X(bar))²].
- **Sample Size (N):** As sample size grows, the denominator expands, driving the standard error SE(b1) toward zero.

The Six Classical Gauss-Markov Assumptions: For OLS estimators to be unbiased, consistent, and efficient, six fundamental assumptions must hold:

- **Assumption A1 (Linearity in Parameters):** , Y = $beta_0$ + $beta_1$ * ln(X) + u is validly linear in parameters).
- Assumption A2 (Zero Conditional Mean of Errors): E(u(t) | X(t)) = 0.
- Assumption A3 (Homoscedasticity / Constant Variance): Var(u(t) | X(t)) = sigma².
- Assumption A4 (No Autocorrelation / Serial Independence): Cov(u(i), u(j) | X) = 0 for all i ≠ j.
- Assumption A5 (Absence of Perfect Multicollinearity): No exact linear relationship exists among the explanatory variables in a multivariate model.

Multivariate Asset Pricing Applications in Finance: In financial economics, multiple regression is deployed to estimate systematic risk exposures, test factor pricing models, and decompose asset returns into alpha and beta components.

Empirical CAPM Estimation Case Walkthrough: Consider an empirical study estimating the CAPM parameters for an aggressive technology stock (TechCorp) against the benchmark market index over T = 60 monthly returns:

- **Jensen's Alpha (alpha):** 139 (Not Significant)
- **Market Beta (beta):** 000 (Statistically Significant)

Fama-French Multi-Factor Asset Pricing Models: Empirical evidence revealed that market beta alone fails to explain the cross-section of equity returns. Eugene Fama and Kenneth French (1993) expanded CAPM into the Fama-French Three-Factor Model by adding firm size and value factors:

Model Evaluation, Selection, and Diagnostic Testing: Estimating regression models is insufficient without rigorous diagnostic evaluation to verify whether the model is parsimonious, well-specified, and compliant with Gauss-Markov conditions.

Testing for Structural Breaks: The Chow Test: Financial relationships often undergo abrupt structural shifts due to regulatory overhauls, financial crises, or monetary policy regime shifts.

Residual Diagnostic Testing: Violations of Gauss-Markov Assumptions

- **Heteroskedasticity (Non-constant variance):** bull; Breusch-Pagan Test: Regresses squared residuals on explanatory variables.
- **Autocorrelation (Correlated residuals):** bull; Durbin-Watson Test: d = 2*(1 - rho).
- **Multicollinearity (High predictor correlation):** bull; Variance Inflation Factor (VIF): VIF(j) = 1 / (1 - R(j)²).

Summary Synthesis

- **Statistical Moments:** ; Quantifies return distributions, fat tails, leptokurtosis, and crash risks.
- **Jarque-Bera Test:** ; Confirms non-normality of asset returns; guides choice of distribution.
- **Gauss-Markov BLUE:** ; Justifies OLS minimization of RSS as the minimum-variance linear estimator.
- **CAPM Regression:** ; Estimates systematic Beta risk and manager abnormal skill (Jensen's Alpha).

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MODULE III

TIME SERIES ANALYSIS IN FINANCE

Stationarity, Random Walks, and Unit Roots: The concept of stationarity is the cornerstone of time series econometrics. A time series is stationary if its statistical properties do not change over time.

Autocovariance and Autocorrelation Function (ACF): For a covariance-stationary process, the Autocorrelation Function (ACF) measures the linear correlation between observations separated by k time periods:

The Pure White Noise Process: A white noise process e(t) is the fundamental stochastic building block of time series models. A sequence is white noise if: bull; Mean is zero: E(e(t)) = 0 for all t.

The Spurious Regression Hazard (Granger Newbold, 1974): , Apple stock price and agricultural wheat yields in India) are regressed against each other using standard OLS:

- 80).
- 001), falsely confirming a causal relationship where none exists.
- **The Granger-Newbold Warning Rule:** Whenever the regression R-squared exceeds the Durbin-Watson d-statistic ($R^2 > d$), the regression is almost certainly spurious, driven by shared non-stationary stochastic trends rather than economic co-movement.

Trend-Stationary vs. Difference-Stationary Processes

Difference-Stationary Process (DSP)

Testing for Unit Roots: Augmented Dickey-Fuller (ADF) Phillips-Perron: To determine whether a series contains a unit root (phi = 1 in Y(t) = phi*Y(t-1) + e(t)), David Dickey and Wayne Fuller (1979) formulated the unit root test by subtracting Y(t-1) from both sides: Change in Y(t) = gamma*Y(t-1) + e(t), where gamma = phi - 1:

Cointegration and Error Correction Models (ECM): While differencing non-stationary I(1) series eliminates spurious regression, it completely discards vital long-run equilibrium information contained in price levels.

Cointegration in Quantitative Pairs Trading (Statistical Arbitrage): In quantitative hedge funds, cointegration forms the mathematical foundation of Pairs Trading:

- **Constructing the Spread:** , Chevron and ExxonMobil): Spread(t) = Price(A, t) - beta * Price(B, t).
- **Normalized Z-Score Tracking:** Calculate Z(t) = (Spread(t) - Mean(Spread)) / SD(Spread).

The Johansen System Cointegration Framework: , stock index, exchange rate, and interest rate), the Engle-Granger bivariate technique is inadequate because multiple cointegrating vectors may exist.

- **Trace Statistic:** Tests the null hypothesis of at most r cointegrating vectors against the alternative of more than r vectors.
- **Maximum Eigenvalue Statistic:** Tests the null hypothesis of r cointegrating vectors against the specific alternative of r + 1 vectors.

The First-Order Autoregressive Process: AR(1)

Second-Order AR(2) Process Yule-Walker Equations: The AR(2) process is: $Y(t) = c + phi_1 * Y(t-1) + phi_2 * Y(t-2) + e(t)$.

- **Stationarity Triangle:** An AR(2) process is stationary if and only if three joint conditions hold: (1) $phi_1 + phi_2 < 1$; (2) $phi_2 - phi_1 < 1$; and (3) $|phi_2| < 1$.
- **Pseudo-Cyclical Dynamics:** If $(phi_1^2 + 4*phi_2) < 0$, the characteristic roots are complex conjugates, generating damped cyclical wave patterns that model business cycles and interest rate fluctuations.
- **Yule-Walker Equations:** Establishes the relationship between autocorrelation coefficients and AR parameters: $rho_1 = phi_1 / (1 - phi_2)$ $rho_2 = phi_1 * rho_1 + phi_2 = (phi_1^2 + phi_2 - phi_2^2) / (1 - phi_2)$.

Moving Average (MA) Models: While AR models express dependence on past values of the series, Moving Average models express the current value as a linear combination of current and past white noise disturbance shocks.

The Invertibility Condition and Wold Decomposition: An MA process is invertible if it can be equivalently rewritten as an infinite-order autoregressive AR(infinity) process: Y(t) = c - Sum[(-theta)^j * Y(t-j)] + e(t).

Autoregressive Moving Average (ARMA) Seasonal ARIMA (SARIMA): George Box and Gwilym Jenkins demonstrated that combining autoregressive and moving average terms creates parsimonious models that require far fewer parameters than pure AR or MA models.

- **AR(p):** ; Cuts off sharply after lag p (zero for all lags > p).
- **MA(q):** ; Tails off (infinite geometric or damped sinusoidal decay).
- **ARMA(p, q):** ; Tails off (geometric decay beginning after lag p - q).

The Box-Jenkins Methodology for ARIMA Modeling: Box and Jenkins (1970) formulated a systematic four-stage iterative methodology for constructing Autoregressive Integrated Moving Average (ARIMA(p, d, q)) models, where d represents the order of differencing required to achieve stationarity.

Step-by-Step Empirical ARIMA Case Study: Consider an econometrician modeling the monthly 10-year Indian Government Bond Yield over T = 120 months:

- **Identification:** 05), confirming non-stationarity.
- **ARIMA(1, 1, 0):** 421 (White Noise)
- **ARIMA(0, 1, 1):** 285 (White Noise)
- **ARIMA(1, 1, 1):** 512 (White Noise)

Out-of-Sample Forecast Evaluation Metrics: To evaluate how well an econometric forecasting model performs on future data untouched during estimation, researchers compute formal predictive metrics across an evaluation test window of length h:

The Hurst Exponent (H) and Rescaled Range (R/S) Analysis: 50.

- **H = 0.50 (d = 0):** The series is an independent random walk (Brownian motion).
- 50 The series exhibits persistent long memory and trend-reinforcing behavior.
- 00 ≤ H The series exhibits anti-persistence and strong mean-reversion.

Practical Time Series Implementation in R and Python: Empirical econometricians deploy script-driven automated pipelines to execute unit root tests, determine optimal lag lengths, and forecast ARIMA models:

MODULE IV

FINANCIAL VOLATILITY MODELING

Volatility Concepts, Measurement, and Stylized Facts: In financial economics, a crucial conceptual distinction exists between Risk and Volatility. While risk broadly encompasses the potential for financial loss or deviation from an expected outcome, volatility specifically quantifies the dispersion or standard deviation of asset price returns over a defined temporal horizon.

Measurement Paradigms: Historical, Implied, and Realized Volatility: Financial econometricians estimate asset volatility using four distinct methodological frameworks:

- **Historical Volatility (Close-to-Close):** ; Backward-looking benchmark.
- **Extreme Value Range Estimators:** Incorporates intraday High (H(t)), Low (L(t)), Open (O(t)), and Close (C(t)) prices.
- **Realized Volatility (RV):** , 5-minute intervals): RV(t) = Sum(r(t,i)²) for i = 1 to M intervals.
- **Implied Volatility (IV) The VIX:** Forward-looking volatility backed out by inverting the Black-Scholes-Merton option pricing formula using observable market option premiums.

The Core Stylized Facts of Financial Volatility: Empirical asset prices display four universal volatility phenomena that motivate conditional volatility modeling:

- **1. Volatility Clustering (Mandelbrot, 1963):** Volatility is temporally dependent.
- **2. Mean Reversion of Volatility:** While volatility clusters in the short run, it does not explode to infinity nor collapse permanently to zero.
- 3.
- **4. Volatility Persistence / High Memory:** Shocks to volatility decay very slowly over time, exhibiting near-integrated properties where persistence parameters approach unity.

The Autoregressive Conditional Heteroskedasticity (ARCH) Model: Prior to 1982, econometric models treated non-constant error variance (heteroskedasticity) as a statistical nuisance to be corrected via Weighted Least Squares.

Parameter Constraints and Unconditional Variance in ARCH(1): For the basic ARCH(1) specification (sigma^2_t = omega + $alpha_1$ * e^2(t-1)):

- **Non-Negativity Constraints:** Because variance represents a squared physical quantity, it can never be negative.
- **Stationarity Condition:** The conditional variance process is covariance-stationary if and only if $alpha_1$.
- **Unconditional Long-Run Variance:** Taking expectations across all periods yields: sigma² = E(sigma^2_t) = omega / (1 - $alpha_1$).
- **Kurtosis and Leptokurtosis:** 0, the unconditional distribution of the ARCH return e(t) has kurtosis: K = 3 * (1 - $alpha_1$²) / (1 - 3*$alpha_1$²).

Practical Limitations of Pure ARCH Models: While conceptually revolutionary, pure ARCH(q) models suffer from three severe practical limitations in empirical finance: bull; Excessive Parameterization: Capturing the long memory of market volatility typically requires 8 to 20 lagged squared residuals (q = 8 to 20), depleting degrees of freedom.

The Generalized ARCH (GARCH) Model: To overcome the excessive parameter requirements of ARCH models, Tim Bollerslev (1986) developed the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework by incorporating lagged conditional variances into the variance equation.

Operational Constraints and Volatility Persistence

- **Non-Negativity Conditions:** To ensure that sigma^2_t is strictly positive for all time periods: omega > 0, $alpha_1$ ≥ 0, and $beta_1$ ≥ 0.
- **Covariance Stationarity Condition:** The GARCH(1, 1) process is covariance-stationary if and only if the Sum of of the ARCH and GARCH parameters is strictly less than unity: ($alpha_1$ + $beta_1$) .
- **Volatility Persistence:** The Sum of ($alpha_1$ + $beta_1$) represents the degree of persistence in volatility.
- **Half-Life of a Volatility Shock:** 50) / ln($alpha_1$ + $beta_1$) (measured in trading days).
- **Unconditional Long-Run Variance (V(L)):** V(L) = E(sigma^2_t) = omega / (1 - $alpha_1$ - $beta_1$).

Estimation via Maximum Likelihood QMLE: GARCH parameters are estimated by maximizing the sample log-likelihood function. Under standard Gaussian errors:

Nelson's Exponential GARCH (EGARCH, 1991): Daniel Nelson formulated the EGARCH model, modeling the natural logarithm of conditional variance:

News Impact Curves (NIC): Pagan and Schwert (1990) and Engle and Ng (1993) introduced the News Impact Curve (NIC) to visualize how different models translate past return innovations e(t-1) into next-period conditional volatility sigma^2_t, holding all past variance constant at the unconditional variance level V(L):

- **Standard GARCH(1, 1) NIC:** A symmetric parabola centered at e(t-1) = 0.
- **GJR-GARCH(1, 1) NIC:** A piecewise asymmetric parabola.
- **EGARCH(1, 1) NIC:** An asymmetric exponential curve where negative shocks increase volatility exponentially faster than positive shocks.

Advanced Extensions: GARCH-M, APARCH, and Multivariate DCC-GARCH

Asymmetric Power ARCH (APARCH)

Component GARCH (CGARCH)

Volatility Forecasting and Financial Risk Management (Value at Risk): Volatility models are estimated primarily to forecast future market turbulence and calculate regulatory capital reserves under Basel risk frameworks.

Computing Value at Risk (VaR) and Expected Shortfall (ES): , 99%):

Regulatory Backtesting: Kupiec's POF Test Basel Traffic Lights: To verify model validity, financial regulators require commercial banks to backtest daily VaR forecasts against actual trading losses over 252 trading days:

- **Exception (Violation):** Occurs whenever actual trading loss on day t exceeds forecasted VaR_t (Loss_t > VaR_t).
- 01 for 99% VaR).

Summary Synthesis

- **ARCH(q):** ; First model of conditional heteroskedasticity; captures volatility clustering but requires many lags.
- **GARCH(1, 1):** ; Parsimonious infinite-memory volatility modeling; persistence = $alpha_1$ + $beta_1$; industry workhorse.
- **GARCH-M:** ; Explicitly models time-varying risk premiums; links return directly to conditional market volatility.
- **EGARCH(1, 1):** ; Guarantees positive variance without parameter constraints; gamma < 0 captures the leverage effect.

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